ARCHITECTURE OF VOWEL SYSTEMS

Louis ten Bosch

1. INTRODUCTION

The vowel systems of natural languages are frequently supposed to satisfy universal rules. For instance, from a phonological point of view, every vowel system should obey a rule with respect to 'diversity' in some way, e.g. with respect to the vowel features length, frontness, height and rounding. Phonological theories often use these features in order to explain the phonological structure of systems as a function of the number of vowels (Kaye et al., 1985; Stevens et al., 1985). However, most of these theories are abstract and still have a descriptive rather than a predictive character (see e.g. Goldstein, 1983; Ohala, 1983).

In this paper we study the internal structure of vowel systems by means of phonetic principles, rather than phonological ones. This just means that the model merely uses phonetic vowel descriptions such as formant positions, spectral behaviour and vocal tract shapes. Furthermore, we will confine the present discussion to systems without the short/long or the oral/nasal opposition and without diphthongs. These restrictions allow a detailed and specific model evaluation, which should be troublesome in the unrestricted case. In this paper we will present and discuss the results of a preliminary test dispersion model. Bonder (1986) presents an alternative model to contruct vowel systems in a hierarchical way.

We hypothesize natural vowel systems to obey two extra-linguistic principles with respect to their internal structure:

- 1 Sufficiency of the acoustic dispersion. This means that the acoustic distance between any two vowels is sufficiently large, according to the claim of vowel distinction in speech.
- 2 Limitation of the articulatory effort function. This restriction is due to the physiological restrictions on the vocal tract shape, which is measured by the effort function.

These principles have an antagonistic character. Evidently, the first one yields a sufficient distance between any pair of vowels, whereas the second principle just claims an overall upper bound to these distances. In order to testify the above hypothesis, both principles are implemented in a software algorithm which actually constructs vowel systems by considering them as stable solutions of a dynamic system. Next, these generated vowel systems are subject to more phonological considerations. For example, they should obey the structure rules and universals as known in phonetic and phonological literature.

The second section consists of an introduction to the present theory. The third section will deal with the implementation of the theory within the model. The fourth section deals with model results whereas in the fifth section presents an evaluation of the results as well as a final discussion.

2. INTRODUCTION TO THE THEORY

2.1 Diversity of vowel systems

One of the well-known properties of vowel systems is the qualitative and quantitative diversity along the set of natural languages in the world (cf. Crothers, 1978; Maddieson, 1984). The number of vowels in a vowel system (denoted N) appears to range from 3 up to about 20, and roughly one-third of all languages do have a length opposition or a oral/nasal opposition or diphthongs (Maddieson, 1984).

A theory which describes the structure of vowel systems should allow for all sorts of phonetic and phonological 'universals'. Despite the fact that the principles mentioned above directly act upon the structure of vowel systems, they apparently should not fully account for the structure of a specific vowel system without further information. The theory should only deal with global probabilistic information about vowel systems in general, such as the range of N, and the modal value of N (which is equal to 5).

This ZWO-project tries to implement the above mentioned principles (with emphasis on the articulatory one) in the phonetic and, to some extent, phonological theory of vowel systems. In the present model this will be done for static short vowels. The theory will be based on two research concepts: the dispersion theory of Liljencrants and Lindblom (1972) on the one hand, and the relation between phonetic and phonological descriptions of vowel systems on the other (ten Bosch, 1985). Liljencrants and Lindblom (1972) initiated the study of vowel systems from a purely acoustic point of view, by considering a vowel system as a system of points within the acoustic vowel space (formant space) and by simultaneously optimizing all intervowel distances. Very recently an improvement of this model has been suggested by Lindblom (1986). Most of the present dispersion models (Stevens, 1972; Lindblom and Liljencrants, 1972; Crothers, 1978; Disner, 1980, 1983) mostly use acoustic arguments only. Our model tries to combine acoustic arguments with articulatory ones.

Crothers (1978) and Disner (1983) dealt with a more phonologically inspired model, using arguments concerning the internal system balance, whereas Kaye, Lowenstamm and Vergnaud (1985a, 1985b) used a theory of generative phonology within their feature model.

Figure 1 presents an overview of the structure of the present model.

2.2 Methodology

In order to study the structure of vowel systems, we have chosen the following methodological set-up. Both principles are implemented in a software algorithm. This algorithm generates vowel systems that satisfy these principles according to several parameters, by maximizing the acoustic dispersion and minimizing the effort function at the same time. This parameter set is accepted, modified or rejected, depending on the similarity of the model results to the literature data. This comparison constitutes the phonological part in the model evaluation (see 4.3).

At the beginning of this project (end of 1984), the most appropriate expressions for the articulatory effort function and the acoustic dispersion were unknown. After having studied many preliminary results, four options



Fig. 1. Overview of the structure of the present model. In top-down direction the phonetic influence is displayed, at the bottom (in reverse direction) the phonological influence. were chosen for the effort function and two for the dispersion function. Subsequent test sessions consisted in verifying model results while using combinations of these options for several values of N. To accept such an option (in the sense of accepting the chosen articulatory effort function and the acoustic dispersion function), the model results should stand the comparison with natural vowel systems.

The present computer model uses exclusively the two principles mentioned above. Because of the absence of linguistic influence in the model, its output will be denoted as 'physical vowel systems'. Natural vowel systems may diverge from their 'physical state' (quantitatively defined) to their 'linguistic state' (mostly qualitatively defined), as a consequence of all sorts of (socio)linguistic factors. In general, however, one expects a probabilistic relation between the 'modal' (= most frequently occurring) linguistic vowel system and its predicted physical counterpart (see section 4.3). The important goal of the project is in fact to clarify explicitly this relationship.

2.3 Relation phonetics-phonology

From experimental phonology, many rules are known with respect to the qualitative structure of vowel systems (Schane, 1973; Crothers, 1978; Maddieson, 1984; Kaye et al., 1985b). Below we present some of the most relevant rules with respect to the classification of vowel systems.

Universals in vowel systems (Crothers, 1978). Stanford Phonological Archiving Project (SPAP):

- 1 All languages have /i a u/. (*)
- 2 Languages with 4 of more vowels have /y/ or $/\epsilon/$.
- 3 Languages with 5 or more vowels have $/\varepsilon$. Often also $/\varepsilon$.
- 4 Languages with 6 or more vowels have $|\mathcal{A}|$ and either |y| or |e|.
- 5 Languages with 7 or more vowels have /e/, /o/ or /y/, /a/.
- 6 Languages with 8 or more vowels have /e/.

7 Languages with 9 or more vowels generally have /o/.

8 The modal vowel system roughly equals /u/, /o/, /a/, /e/, /i/ (30 %).

9 Short high and low vowels are often more centralized then their long counterparts.

(*) modification according to Disner (1983) UCLA Phonological Segment Inventory Database (UPSID):

1a All languages have /i/, /a/.

1b Languages with 3 or more vowels generally have /i/, /a/, /e/ or /i/, /a/, /o/; with less probability /i/, /a/, /u/.

We attempted to design a method for comparing the phonetic model results (in terms of formant positions) and the phonological data. In section 4.3 we will be able to explain this method in more detail, after having introduced an appropriate function for measuring similarity probabilities.

2.4 Optimization

Several mathematical optimization methods have been applied, the most important one being the well-known gradient method. An optimal vowel system is found by iteratively shifting all vowels in the system simultaneously to a more optimal position. The solution of this optimality problem is in fact found by considering the problem in terms of dynamic systems, of which the fix points represent the (sub)optimal vowel systems. The search is terminated if either a solution is found, no progression can be reached (within a predefined tolerance), in case of a degenerated solution, or in case of excessive CPU time consumption.

3. THE MODEL

3.1 Articulatory constraints

3.1.1 General remarks

Evidently, all natural vowel systems obey certain articulatory constraints. The limitation of the vowel space is a direct consequence of such constraints. Vowels will simply not appear in vowel systems if they are undesirable from an articulatory point of view, implying the effort function being too large in such cases. In order to deal with the vocal tract shape, we use the so-called n-tube model (Stevens, 1972; Atal et al., 1978; Bonder, 1983). In these models the shape of the vocal tract is approximated by a step function which constitutes the segment areas of the n straight segments of the tube.

In the present model n equals 4, which yields a three-dimensional articulation space (the second, third and fourth segment area being independent; the first area, situated near the glottis, equal to unity). The overall tube length is fixed. This 4-tube choice simplifies the calculations, but appears to hinder the evaluation of model results with respect to some specific articulation data, such as the vowel feature 'rounding'.

3.1.2 Defining an effort function

The **articulatory effort function**, denoted d_A , is based upon the shape of the vocal tract during production of the vowel in terms of an 4-tube model (Bonder, 1983, 1986; ten Bosch, 1985). The larger the (weighed) distance between a particular articulatory position and a neutral position is, the more difficult the actual pronunciation of the corresponding vowel is supposed to be and the larger the corresponding effort value will be. It is well known that such a definition for 'articulatory effort' bears several difficuities because of the non-uniqueness of the vocal tract shape with specific F1- and F2-values (cf. Atal et al., 1978; Bonder, 1983; ten Bosch, 1985). We removed this ambiguity in a satisfactory way by applying one extra minimality constraint in terms of the effort function. The effort function measures the 'degree of pronunciability' of tubes. To be more precise, we have to formulate a few methodological claims for d_A . d_A will be called 'adequate' if the following properties hold simultaneously: a the structure of natural vowel systems is satisfactory simulated by d_A for many values of N (up to 9, at least);

- b the boundary of the actual vowel space is directly explained by contour lines of d_A .
- c the expression for dA is independent of n.
- d d_A is non-negative and equals 0 only in case of a neutral articulatory position.

In the present model four different expressions can be used to constitute d_A . These expressions will be denoted without subscript A by d_1 , d_2 , d_3 and d_4 . They all are related to the shape of the vocal tract in a way. The use of several functions d_1, \ldots, d_4 may be justified by considering the possibility of d_A being dependent of more than one articulatory tube feature. For example, d_A might depend on both the tube distance to the neutral straight tube (d_1), and the opening degree (d_3) which measures the area at the lips compared to the area at the glottis.

3.1.3 Expressions for d_A

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 d_A is chosen as a combination of the following four expressions, in which $S_1,\,S_2,\,S_3$ and S_4 denote the four segment areas:

a. articulatory distance d₁ between the tube and the neutral tube:

 $d_{1} = 0.25 * \Sigma (\beta)_{i} * [Si - 1]^{2}$ (1)

 (β_1, β_2) in which $(\beta)_i$ are weighing parameters along the tube;

b. the straightness d_2 of the tube:

 $d_2 = sd[log(S_1/S_2), log(S_2/S_3), log(S_3/S_4)]$

in which sd[number set] denotes the standard deviation of the number set;

(2)

c. the opening degree d_3 of the tube:

 $d_3 = S_4/S_1$ (3)

d. the constriction degree d_4 of the tube:

$$d_{4} = (1/16) * \prod_{i=1}^{4} (S_{i} + 1)/S_{i}$$
(4)

in which 1/16 is a normalisation factor such that $d_4(1, 1, 1, 1)$ equals unity. One may observe that not all of these functions are adequate in the sense described above. Until now, however, other suitable adequate functions have not been found, except perhaps an effort function of a totally different type (see section 5).

The articulatory effort function for a whole vowel system (denoted $D_A(system)$) is defined as the average of all d_A -values within the system:

3.2 Acoustic constraints

3.2.1 General remarks

There exists a strong evidence for acoustic dispersion theory (Disner 1980, 1983). About 90% of all the languages recorded in UPSID (UCLA Phonological Segments Inventory Database) obey a dispersion rule in a broad sense (Disner, 1983). Of course, one should carefully interpret such data. Disner weakens the claims of dispersion theory substantially by introducing 'empty holes'. Nevertheless, one may a priori defend a dispersion theory with arguments dealing with minimization of confusion events in running speech or optimality in the sense of information theory. Those arguments strike more than vowel dispersion only, also consonants should consequently be 'evenly spread'.

3.2.2 Defining the acoustic dispersion

The **acoustic dispersion** within a vowel system is based upon the relative and absolute position of vowels in the formant space. The use of this type of dispersion is inspired by work of Lindblom (Liljencrants and Lindblom, 1972; Lindblom, 1975, 1986). In our approach, this dispersion function relies only on the position of the first two vowel formants.

Although vowel dispersion is easy to claim, it is much more difficult to get an exact expression for it. One may follow the classical set up:

$$d_{F}(v_{i}, v_{i})^{2} = [F_{1i} - F_{1i}]^{2} + [F_{2i} - F_{2i}]^{2}$$
(6)

or, alternatively

$$d_{F}(v_{i}, v_{i})^{2} = [\log(F_{1i}) - \log(F_{1i})]^{2} + [\log(F_{2i}) - \log(F_{2i})]^{2}$$
(7)

in which F1(v) and F2(v) are the first two formants of the vowel v, and $d_F(v_1, v_2)$ stands for the acoustic distance between the vowels v_1 and v_2 . From the theory of hearing it is known that formula (7) is much better defendable than formula (6) (Pols et al., 1973). In the present model the logarithmic formant frequencies (7) are used, with justification from the hearing argument.

Instead of using (6) and (7), one may plot (F_1, F_2-F_1) or $(\log(F_1), \log(F_2/F_1))$ and then define dF as the ordinary euclidian distance between these latter points. However, this option appeared to give highly uninterpretable results. Other types of distances, such as spectral-based ones, need not to be considered because of the use of formant-based vowel descriptions here.

The difference between the logarithmic and linear $d_{\mathbf{F}}$'s are small with respect to a small vowel neighbourhood. In other words, the local geometry does not change. Globally however, the geometry of the vowel space is changed, and the boundary of the vowel space does change accordingly. This is relevant because the model and therefore its solutions have in fact a geometrical character. The influence of the global shape of the vowel

space in the Liljencrants and Lindblom model will be evident to anyone acquinted with this model.

3.2.3 Perceptual confusion errors

From a methodological point of view, it might be elegant to relate the acoustic dispersion within a vowel system with its perceptual confusion matrix, instead of defining dispersion in terms of dF itself. This argument may be well-reasoned as follows. Firstly, dF itself has no easy perceptual interpretation without referring to either ear models or confusion errors in (running) speech. Secondly, a relation with confusion matrices simplifies reference to experimental data, provided these data do not contain much linguistic information relative to the proper acoustic information. This yields at least the possibility of an empirical verification for the expression related to vowel dispersion. Thirdly, one introduces a probabilistic part within the theory as a parallel with the probabilistic part in phonetic and phonological statements concerning natural vowel systems.

The present expression relating the **confusion chance** (denoted $p(v_1, v_2)$) between two vowels v_1 and v_2 and their acoustical distance (denoted $d_F(v_1, v_2)$) reads

$$p(v_1, v_2) = \exp(-\alpha^* d_F(v_1, v_2))$$
(3)

in which α denotes a positive normalisation factor, and 'exp' the exponential function. The formula implies that the **discrimination chance** between v₁ and v₂, which is by definition equal to $1 - p(v_1, v_2)$, equals $1 - exp(-\alpha * d_F(v_1, v_2))$.

Consequently, the overall discrimination chance (denoted $D_F(system)$) in the vowel system equals $\Pi(1 - p(v_i, v_i))$.

In this expression denotes ' Π ' the product over all vowel pairs $(v_{i},\,v_{j})$ in the system.

Formula (8) defines a **probability matrix** corresponding to the intrinsic structure of natural vowel systems. This probability matrix should not be confused with the labeling matrix that is extracted from vowel identification experiments. The above correspondence constitutes a well-defined mapping from the set of vowel systems into the set of real numbers. $D_F(system)$ has the following relevant properties:

- $-0 < D_F(system) < 1;$
- D_F(system) tends to 0 if some vowel v_i gets close to some vowel v_i;
- $D_F(system) = 1$ in the ideal case of perfect intelligibility of any vowel in the system.

These properties allow the interpretation of $D_F(system)$ as a well-defined intelligibility probability function.

Expression (8) will certainly not hold if oppositions of other type than formant oppositions occur in the vowel system. For instance, the duration may play an important role in the discrimination. Therefore short/long oppositions must be excluded in the present consideration. Consequently, we have to conclude that the model results can now only be adequately compared with data of natural vowel systems without long/short opposition. We will examine this interesting fact in section 5. Figure 2 presents a formulae scheme as used in the present model.

3.3 Optimalization technique

Let x be a point in some euclidian space of arbitrary (but finite) dimension. Many optimalization techniques are available in order to optimize a sufficiently differentiable, positive function g(x) with boundary condition h(x) = c. One of them consists in optimizing $r(x) = g(x)^2 + S \cdot (h(x) - c)^2$, in which S is a slack variable (very large, arbitrary). This method is implemented in the model as follows. Q will be the overall optimization parameter which is to be minimized. The expression for Q reads

$$Q = D_A(system)^2 + S * (D_F(system) - 1)^2.$$
 (9)

Minimization of Q implies minimization of $D_A(system)$ with $D_F(system) = 1$ as a boundary condition, in other words, minimization of D_A without any system confusion.

4. RESULTS OF THE MODEL

4.1 Background

With the computer algorithm SYSGEN which we developed, over 250 vowel systems were generated which minimize both the articulatory effort and the overall confusion chance for different combinations of d_F and d_A . Furthermore, a tentative evaluation was implemented in the program. This evaluation will be described in section 4.3.

For reference, we show the vowel space in two stylized versions (figure 3). One version is roughly indicated by two solid lines which represent contour lines of a special effort function, belonging to different specific function values. We will deal with this function in section 5. The grey area is empirically determined and plotted such that it covers the position: of nearly all natural vowel systems.

4.2 SYSGEN results

In figure 4, we show the model results for N = 3. Using the four expressions d_1 , ..., d_4 mentioned in paragraph 3.1.3, only one combination of these four expressions for d_A leads to acceptable results. This combination reads $d_A = 0.5*d_1 + 5*d_2 + 5*d_3 + 5*d_4$. For the time being we will deal with this combination only.

Figures 5 and 6 show the solutions in the case N = 5, N = 6 respectively. In figure 7, we show the tendency in the positions of vowel systems for higher N which already can be extracted from the cases N = 5 and N = 6. These results clearly show that the formulae used so far are inadequate for the description of natural vowel systems. However, these results are instructive with respect to the dependence of vowel systems on several options for d_A and d_F . (In section 5 we will deal with several possible improvements.)

4.3 SYSGEN - phonetics versus phonology

Figure 8 shows the results of the comparison between the model solutions with the phonological data from the UPSID database (Maddieson, 1984). We tried to evaluate the present model as follows.

Firstly, all phonologically specified characters such as /a/, /e/ etc. were assigned to specific formant positions. Because of the phonetic underspecification of the phonological data, the comparison should have a probabilistic character.

Secondly, we evaluated the similarity of the model system to the **phonetic** translation of the phonological reference system. We constructed the so-called minimal pairing between both systems or subsets of them, i.e. we looked for the 'cheapest' one-to-one correspondence. Next, we defined the confusion probability conform formula (8),

$$p(v_{i}, v_{i}') = \exp(-\alpha * d_{F}(v_{i}, v_{i}'))$$
(10)

(in which v_i and v_j ' denote a vowel from the model system and the reference system respectivily), and the 'similarity probability' (denoted SP) by

$$SP = II p(v_i, v_i')$$
(11)

in which ' Π ' stands for the product over all appropriate pairs (i, j). SP depends on the relative positions of the model system and the reference phonological system only.

Thirdly, we evaluated the SP-value between the predicted N-vowel system and all the phonological systems from UPSID which lack long/short-opposition as well as diphthongs for N from 3 up to 9 (figure 8).

One observes the rapidly declining fit of the model for increasing N. The heavy line connects the maxima of the SP-values for each N. The thin lines show the ramification of several possibilities for several values of N. In this comparison only vowel systems without any length opposition and without diphthongs were used, in order to evaluate the results in a plain way. In the UPSID database, which contains 317 languages from all over the world, 246 languages fulfil these constraints. In figure 9 we show some information of the database (Maddieson, 1984). (a) shows the fraction of this subset of languages with a specific number of vowels. (b) simply represents the fraction of all 317 languages with a specific number of vowels. (c) shows the fraction of 317 all languages with a specific number of vowel qualities. Figure 9 shows that the subset without length-opposition or diphthongs possesses a more specific distribution of the number of vowels.



Fig. 2. An overview of the formulae as used in the model. For explanation of all the symbols see the text.



Fig. 3. This figure shows the vowel space (grey area) and two contours of a special articulatory effort function, the upper and lower one corresponding to a realistic and an extreme effort value, respectively. See also the discussion in section 5. The straight line represents the F1=F2 axis.



Fig. 4. The predicted 3-vowel system (SYSGEN).



Fig. 5. Predicted 5-vowe! system (SYSGEN).







Fig. 7. Tendency of the position of the cardinal vowels in systems as predicted by SYSGEN. This figure shows the inadequacy of the present articulatory effort function d_A .



Fig. 8. Measure of agreement (SP-values) between the preliminary model and the phonological descriptions of a subset of the UPSID-database (see the text). The figure shows the probability of the predicted vowel systems being similar to phonological systems versus the number of vowels. The heavy line (a) connects all the found maxima. The thin lines (b) show a part of the possible ramifications.



Fig. 9. Relative occurrence of languages in a specified database versus the number of vowels (a, b) or vowel qualities (c) (from Maddieson, 1984). For an explanation see section 4.3.



Fig. 10. The vowel space indicated by a set of contour lines of the alternative articulatory effort function d_A . The ellipse (the small closed contour) contains the schwa as its central point. The lowest contour defines already fast unrealizable articulatory positions.

5. EVALUATION OF SYSGEN RESULTS

The present model is not convincing for N > 5. The main advantage now with respect to older models is the explanation of the boundary of the vowel space by an articulatory effort function, instead of defining the boundary outside the structure-defining section in the model. Close inspection of the generated model data has led to several propositions for improving the model. Although the correctness of the model is still under discussion, slight modifications seem already to be able to overcome the most striking irregularities.

1. The expansion of the calculated vowel systems for N > 5 is partly due to the structure of the expression $D_A(system)$. $D_A(system)$ is defined as the average over all vowels of d_A . That implies that vowels outside the vowel space are 'permitted' as soon as the other ones contribute less to the overall sum. In a subsequent model this will be modified by introducing the exponent p > 1 in the expression

$$d_{A}^{P} = \sum_{i=1}^{N} (d_{A})^{P}(v_{i})$$

The limiting case $p = \infty$ yields another concept for d_A , which merely simulates an articulatory box principle. It implies that $D_A(system)$ equals $\max(d_A(v_i))$. All vowels in the volume $[v; v \text{ such that } d_A(v) < D_A(system)]$ are permitted without new contributions to $D_A(system)$, whereas the boundary of the vowel system is still not fixed, namely given by $[v; d_A(v) = D_A(system)]$, while $D_A(system)$ is subject to the minimization procedure in the iteration phase. The scope of this argument will be examined further in a forthcoming paper.

(12)

2. There is another way to introduce an articulatory effort function in the dispersion theory. This alternative allows the interpretation of d_A in the same probabilistic way as D_F . The alternative d_A is defined as the minimum of an effort function along all tubes with the same formant position. The advantage of this alternative function consists in the recent discovery that the boundary of the vowel space is just described as one of the contour lines of this function. This property exactly fulfils one of the a priori claims for the adequacy of d_A . In figure 10 we show the structure of the set of contour lines of this alternative effort function. Two of these contour lines have already been plotted in the figures 3 until 7.

3. The model results achieved so far suggest that the structure of vowel systems may be well described by means of two antagonistic principles concerning both the articulatory and the perceptual properties of vowels. This description is not complete in the sense of being able to predict exactly the positions of all the vowels in a particular system, but it rules merely the global probabilistic structure of systems containing stable vowels. It is now well worth considering the possibility that the articulatory constraints only rule the outer regions of the vowel space (e.g. the determination of the boundary of the vowel space), whereas the perceptual claim D_F merely organizes the internal structure of vowel systems. The most difficult part will consist in the improvement of the model with respect to d_A .

In a forthcoming paper we will deal with refinements of d_A and other model improvements which have already been suggested above, as well as the implementation of the long/short opposition and diphthongs.

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